

An opaque selling scheme to reduce shortage and wastage in perishable inventory systems

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ABSTRACT

Effective management of perishable inventory systems is often strewn with challenges, especially when a strong trade-off relationship exists between shortage and wastage of perishables: A smaller inventory increases the chance to lose sales (leading to higher expected shortage cost), while a larger inventory increases the chance to waste perishables (leading to higher expected wastage cost). The root cause of this strong trade-off relationship is high product demand variability. To mitigate the issue and reduce the cost of operating perishable inventory systems, some grocery stores utilize an opaque selling scheme: selling an anonymous product whose brand or exact specification is shielded from customers at the time of sale. The use of opaque products has now become a popular means to reduce shortage/wastage at grocery stores. However, there has been little discussion of the effectiveness of opaque schemes applied to perishable inventory systems.

In this paper, we propose an opaque scheme based on the balancing policy on demand, which tries to average out orders for products. We confirm both analytically and numerically that this opaque scheme effectively reduces product demand variability, thereby reducing both shortage and wastage for perishable inventory systems under a base-stock policy. We also present an analytical formula that reveals insights into the opaque scheme: The ratio between the opaque proportion and the coefficient of variation of product demands plays a key role to determine the effectiveness of our opaque scheme. Furthermore, we provide a rule of thumb to find the threshold opaque scheme parameters needed to achieve the maximum total cost savings for perishable inventory systems. We hope that many retailers selling perishables (e.g., fresh produce and baked goods) find the opaque scheme useful, implement it, and contribute to the reduction of the food wastage.

1. Introduction

In 2011, about one-third of food produced for human consumption was wasted globally [1]. In the United States in 2010, about 133 billion pounds (31%) of the food supply was wasted, including 43 billion pounds at retail level [2]. This large amount of food waste not only imposes an economic loss for producers and sellers, but also exacerbates food insecurity for low-income communities [3]. One of the contributors to food waste is poor management of perishable inventory: [2] point out that food waste at retail level occurs from “overstocking or overpreparing due to difficulty predicting number of customers”. As is commonly known, “businesses are frequently motivated to overproduce or over-order because they are afraid of running out of food or failing to meet client expectations” [4]. For retail store managers, overstocking perishable products is necessary in anticipation of the possibility of lost sales when product demand is highly variable. However, overstocking

leads to higher expected wastage. The root cause of this trade-off relationship is high variability of product demand. If the product demand was accurately predictable with no uncertainty, a seller would order the exact amount necessary and yield no lost sales (i.e., shortage) or wastage; however, an accurate prediction of product demand is not always possible.

To cope with highly variable product demand in perishable inventory systems, retail store owners often utilize an opaque selling scheme (or simply, an *opaque scheme*). This scheme involves the sales of an opaque product, wherein customers only know the types of products they may possibly get, but do not know the exact product attributes until they purchase or receive it. The simplest example would be the following. Suppose a seller receives an order of 30 red apples, 50 yellow apples, and 20 *opaque* apples; the seller allocates 20 opaque orders to

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red apples, making demands of both red and yellow apples equal to 50, so that the seller can average out red and yellow apples' demands and achieve less variability for both product demands. Many merchants and producers implement a variety of opaque schemes successfully without calling them *opaque*: for example, a package of assorted food items (such as bagels and donuts), whose combination is not easily identified in a brown bag, is often sold at a discount price at supermarkets; Apple Japan sells Lucky Bags that contain various currently-sold merchandises in sealed bags at a discount price; Priceline sells anonymous hotels as “Express Deal” together with hotels that are fully specified. Another example is the app, “Too Good To Go”, which aims to reduce food waste by selling unspecified food items (e.g., bakeries, delis, and cakes). The number of mobile apps that implement various opaque schemes has rapidly grown in recent years. In addition, the COVID-19 pandemic has driven customers to shift from physical to online stores across many categories such as fresh produce and groceries, making the implementation of an opaque scheme easier than before.

1.1. Contributions and outline

The benefit of opaque schemes for non-perishables (e.g., clothes) has been observed in practice and discussed in the literature. However, even though we observe opaque schemes being utilized at many grocery stores and bakeries selling fresh foods through mobile apps, a quantitative analysis of opaque schemes for perishables has not been published. Thus, the research questions we address in this paper are: (i) how we can evaluate the effectiveness of an opaque selling scheme in a simple mathematical form; (ii) under what conditions does our opaque scheme provide benefits in perishable inventory systems; and (iii) what recommendations our study can provide to retail store owners managing to reduce wastage. This paper helps practitioners in addressing these questions by introducing an opaque scheme suitable for perishables, evaluating its effectiveness for perishable inventory systems, and providing simple rules of thumb for practitioners when implementing our opaque scheme.

The remainder of this paper is structured as follows. We review the previous literature and explain the research gap in Section 2. We provide exact/approximate formulas to obtain the variability (Proposition 1, Corollary 2) and correlation (Lemma 2) of product demands under our opaque scheme in Section 3; we provide an approximate formula to obtain the threshold variability to make the expected shortage and wastage close to zero (Proposition 2) in Section 4; we confirm the accuracy of the analytical formulas using numerical experiments in Section 5; and we discuss the managerial insights into the opaque scheme in Section 6. Finally, Section 7 summarizes our study. The list of notations used in this paper and all proofs are in the Appendix.

2. Related literature

To reduce the amount of wastage, many retail store owners often sell their about-to-expire products at significantly discounted prices. This practice may not be desirable; knowledge that there may be a last-minute discount could dissuade buyers from purchasing items at regular (higher) prices. Reinforcing such a buyer habit may decrease the average selling price and seriously damage the market [5]. Another common practice is to donate older products to food banks, which certainly contributes to the reduction of wastage; however, retail stores still pay for wastage. The root cause, high variability of product demand, still remains.

2.1. Key idea: risk pooling

To cope with risk (volatile and unpredictable customer demand), operations managers frequently use a scheme called *risk pooling*—a strategy to suppress the variability using pooled demand (combined demand). Risk pooling has been proven effective in the context of

supply chain management (see, e.g., [6–8]). An opaque scheme is one such example: It utilizes pooled demand as the means to control highly variable and unpredictable demands of products. This scheme has attracted much attention especially in revenue management [9–14]. For example, Priceline has reduced shortage (lost sales) and wastage of hotel rooms by offering opaque hotel rooms to their customers. Under Priceline’s opaque scheme, room buyers do not know the brand of the hotel prior to purchase; however, they may reserve an opaque hotel room (at a discount room rate) that would stay unoccupied and be wasted if Priceline did not offer opaque hotel rooms. [13] reveals the relationship between opaque hotel room pricing and hotel room inventory, thereby confirming the effectiveness of Priceline’s opaque scheme.

2.2. Research gap

Although an increasing attention is paid to opaque schemes in revenue management, the literature on opaque schemes in inventory systems is still very small. Table 1 lists papers that discuss opaque selling schemes in revenue management and in inventory management. [15–17] study an effectiveness of opaque schemes when selling non-perishable products. [15] focus on the pricing schemes of opaque products when pessimistic and risk-neutral customers exist. [16,17] discuss the opaque scheme for non-perishable inventory systems following a continuous-review $(0, S)$ policy, where an order is placed to make the inventory level back to the order-up-to level S when the inventory is depleted. An opaque scheme is applied to the system with two non-opaque products [16] or more [17]; their theoretical analysis is limited to the no lead time, backlogging, or lost sales case.

These existing studies focus on non-perishable products (e.g., clothes), not on perishable products (e.g., fresh produce, baked goods). As far as authors know, theoretical analysis of opaque schemes has not been made for perishable inventory systems, despite the fact that many retail stores already implement such schemes. Our study fills the research gap by looking into the effectiveness of opaque schemes in perishable inventory systems. We apply the no lead time assumption for our study in accord with previous opaque scheme studies. Our paper has two major differences from existing studies on non-perishable inventory systems: (1) we consider a periodic base-stock policy, which is more suitable for perishable products; and (2) our opaque order fulfillment procedure is based on the number of orders received in each period (BPD) and not based on the on-hand inventory (BPI).

2.3. BPI (balancing policy on inventory)

To carry out the risk pooling principle in a perishable inventory system, we can consider *balancing policy on inventory* (BPI): Using pooled demand (opaque product demand), BPI attempts to average out the inventory level among products. The actual procedure to implement BPI would be to “fulfill demand for opaque products by using the product with the highest on-hand inventory level” [17]. For non-perishable inventory systems, BPI is often considered as an optimal policy [18]; furthermore, even a small proportion of opaque product demand can effectively improve the operation [17]. However, sellers may face challenges when implementing BPI in perishable inventory systems: First, larger on-hand inventory often implies larger amount of aged (about-to-perish) inventory. Thus, BPI may implicitly prioritize the use of older units to fulfill opaque demand whenever possible. When sellers implement BPI, customers might regard opaque products as not only *opaque*, but also *aged*. (For example, Lucky Bags sold at Apple Japan, a popular opaque selling scheme, seldom contain recently-released products. Instead, we often observe a bag full of obsolete products since sellers implement BPI.) Such a perception harms the reputation of stores and provides a disincentive for customers to purchase opaque products unless they are sold at hugely discounted prices. Second, the proper execution of BPI for perishable inventory

Table 1
Studies discussing opaque selling schemes.

Paper	Management area (focus)	Scope
Fay [9]	Revenue (travel industry)	Opaque goods sold through intermediaries
Jiang [10]	Revenue (travel industry)	Price discrimination and market segmentation
Fay and Xie [11]	Revenue (travel industry)	Probabilistic selling
Jerath et al. [12]	Revenue (travel industry)	Last-minute selling vs. opaque selling
Anderson [13]	Revenue (travel industry)	Priceline's opaque selling scheme
Anderson and Xie [14]	Revenue (travel industry)	Optimal opaque pricing policy
Elmachtoub and Hamilton [15]	Revenue (non-perishables)	Pessimistic vs. risk-neutral customers
Elmachtoub et al. [16]	Inventory (non-perishables)	Two products, opaque selling vs. dynamic pricing
Elmachtoub et al. [17]	Inventory (non-perishables)	Two or more products, balancing policy on inventory
Present study	Inventory (perishables)	Balancing policy on demand, perishable inventory system

is extremely complicated; for example, it is hard to determine which inventory to use first for fulfilling demand for opaque products, smaller inventory of older products vs. larger inventory of newer products. These challenges do not exist for non-perishables; however, they may become a major obstacle when implementing BPI for perishables.

2.4. BPD (balancing policy on demand)

We propose a simpler alternative, *balancing policy on demand* (BPD), which attempts to average out product demand. BPD fulfills demand for opaque products by using the product with the least demand. BPD does not utilize on-hand inventory information, and thus may have inferior performance compared to BPI; however, we show that BPD only requires a small amount of opaque product demand to average out demands among products, thereby achieving a balanced inventory level that BPI also achieves. Furthermore, BPD does not suffer from two challenges that BPI faces: First, BPD is simpler to implement in practice than BPI; moreover, under BPD, a seller does not necessarily allocate an older inventory to fulfill the demand of opaque products. BPD requires a purchaser of opaque products to give up only the *specificity* of products (in exchange for some monetary or nonmonetary benefits for customers), not the *quality* (i.e., freshness, age) of products. We show in this paper that our opaque scheme using BPD effectively reduces the variability of demands and achieves high cost savings for perishable inventory systems.

2.5. Scaled Poisson demand for non-opaque products

BPD opaque schemes (as well as BPI) can be applicable to any product demand distributions (regardless of perishables or non-perishables). For the sake of analytical studies, we assume that a periodic demand of each product follows a scaled Poisson distribution. An alternative, more commonly used demand distribution in inventory models would be a compound Poisson distribution¹ (see, e.g., [19,20] and references therein); however, we utilize a scaled Poisson distribution since it can accurately approximate various compound Poisson distributions [21] and yet, is simpler to implement than a compound Poisson distribution. Throughout this study, we allow demands and orders for inventory replenishment to be non-integers (note: examples of non-integer perishables are cut vegetables/fruits, which are gaining in popularity recently).

2.6. Base-stock model for perishable inventory systems

The optimal inventory policy for a perishable inventory system is hard to find due to *curse of dimensionality*—the intractability of all possible change of inventory items with different ages over infinite period. Thus, various heuristic methods have been proposed (for a

review, please see [22,23]; one of the examples is [24]). Among many heuristic policies that are available today, a simple base-stock policy is still considered as an effective scheme [25–27]. In this study, we use this simple base-stock policy. We further impose simplifying assumptions used in [28]: fixed shelf life, i.i.d. (independent and identically distributed) demand, periodic review, FIFO (first-in first-out) issuance policy, and the total cost that involves shortage and wastage costs. The analysis of the models with more relaxed assumptions would be the next step after this study.

3. Opaque selling scheme

This section introduces our (n, p) opaque scheme based on BPD policy. The result we obtain in this section applies to both perishable and non-perishable products. The goal of the (n, p) opaque scheme is to reduce the variability of non-opaque product demands by allocating opaque product demand to non-opaque product demands (i.e., fulfilling opaque product orders using non-opaque products.) Notations used in this study are summarized in [Appendix A](#)

3.1. Model

The (n, p) opaque scheme is characterized by n (≥ 2) non-opaque products and probability vector $p := (p^1, p^2, \dots, p^n)$, where each $p^i \in [0, 1]$ represents the probability that a customer for non-opaque product $i \in \{1, \dots, n\}$ switches to purchase an opaque product if both opaque and non-opaque products are offered to customers. We introduce three random variables to describe the demands at various stages: original demands (D_0^i for non-opaque product i), intermediate demands (X_p^i for non-opaque product i and X_p^0 for an opaque product), and adjusted demands (D_p^i for non-opaque product i). *Original demands* are the number of orders placed for non-opaque products when no opaque product is offered to customers. We represent original demands by independent (but not necessarily identical) random variable D_0^i for product i , whose mean is $\mu^i = \mathbb{E}[D_0^i]$ and variance is $\sigma_i^2 = \mathbb{E}[(D_0^i - \mu^i)^2]$. *Intermediate demands* are the number of orders placed for non-opaque and opaque products when both non-opaque/opaque products are offered to customers. We represent intermediate demands by X_p^i for non-opaque product i and X_p^0 for an opaque product. Finally, *adjusted demands* are the number of orders for non-opaque products including some of the orders originally placed for an opaque product. (This adjustment is essentially the procedure to distribute opaque demand X_p^0 to n non-opaque demands $X_p^i, \forall i$.)

Using these variables, we can describe the procedure to implement our opaque scheme under BPD:

Step 1: (status quo) All products sold are non-opaque, and their demands are D_0^i (original demand).

Step 2: (implementation) An opaque product is now sold. All non-opaque product demands change from D_0^i to X_p^i (intermediate demand). In addition, an opaque product shows its demand X_p^0 .

Step 3: (adjustment) The entire opaque demand X_p^0 is distributed and added to $X_p^i, \forall i$. Through this adjustment, we obtain D_p^i (adjusted demand).

¹ A compound Poisson distribution is versatile and can replicate various consumer purchasing behaviors: Each customer arrives following a Poisson process, whose purchases follow an arbitrary distribution.

3.2. Assumptions

In order to facilitate the analysis, we make a few simplifying assumptions for the model. First, we require the average of adjusted demand (orders) remain unchanged: $\mathbb{E}[D_0^i] = \mathbb{E}[D_p^i]$ ($= \mu^i, \forall p, \forall i$). Second, we assume that the total demand (total number of orders) remains unchanged after an opaque product is offered to customers: $\sum_i D_0^i = \sum_i X_p^i + X_p^0 = \sum_i D_p^i, \forall p$. Lastly, we assume that the customers' decisions are independent from each other: $\mathbb{E}[X_p^i] = (1 - p^i)\mu^i$ and $\mathbb{E}[X_p^0] = \sum_i p^i \mu^i$. Thus, we can interpret p^i as a proportion of orders shifted from non-opaque product i to an opaque product under the opaque scheme: $p^i = 1 - \mathbb{E}[X_p^i]/\mu^i$. A seller can control the proportion p by providing customers incentives to purchase an opaque product. Examples of such incentives include discounts, reduced/free shipping fees, and labeling an opaque product as an eco-friendly choice that helps reduce food waste. If no customers are inclined to purchase an opaque product, then $p = \mathbf{0} = (0, 0, \dots, 0)$ (no opaque case), and thus $X_0^0 = 0$ and $D_0^i = X_0^i = D_0^i, \forall i$. In contrast, if all customers always purchase an opaque product, then $p = \mathbf{1} = (1, 1, \dots, 1)$ (full opaque case), and thus, $X_1^i = 0, \forall i$ and $\sum_i D_0^i = X_1^0 = \sum_i D_1^i$. For simplicity of notation, if all elements of vector p are equivalent, we denote a subscript as scalar p unless it creates a confusion (e.g., we denote D_0^i instead of D_0^p).

3.3. Variance $\sigma_{n,p}^2$

Let the average variance of original demands be $\sigma^2 := \frac{1}{n} \sum_i \text{var}(D_0^i)$ and the average variance of adjusted demands under the (n, p) opaque scheme be $\sigma_{n,p}^2 := \frac{1}{n} \sum_i \text{var}(D_p^i)$. The following lemma shows the bound of $\sigma_{n,p}^2$.

Lemma 1. *The average variance $\sigma_{n,p}^2$ is bounded as follows. $\sigma_{n,p}^2$ takes an upper bound $\sigma_{n,0}^2 = \sigma^2$ when $p = \mathbf{0}$ and takes a lower bound $\sigma_{n,1}^2 = \frac{\sigma^2}{n}$ when $p = \mathbf{1}$.*

$$\sigma^2 \geq \sigma_{n,p}^2 \geq \frac{\sigma^2}{n}. \tag{1}$$

Lemma 1 implies that $\sigma_{n,p}^2$ is minimized at $p = 1$ when the deviation of each product demand from its mean is averaged out among all products: $D_1^i = \mu^i + \frac{1}{n} \sum_j (D_0^j - \mu^j)$. We can establish the opaque product demand allocation policy (BPD) as Corollary 1. We omit the proof, which follows straightforwardly from the fact that $\sigma_{n,p}^2$ is minimized when we minimize the term $\sum_{i < j} \left((D_p^i - \mu^i) - (D_p^j - \mu^j) \right)^2$ (see the proof of Lemma 1).

Corollary 1. *Balancing policy on demand (BPD): To minimize the average variance $\sigma_{n,p}^2$ under the (n, p) opaque scheme, we distribute opaque product demand (X_p^0) to non-opaque product demands $(X_p^i, \forall i \in \{1, \dots, n\})$ with smallest $X_p^i - \mu^i$ or distribute equally if there is a tie.*

For the rest of this paper, we impose additional conditions to obtain analytical expressions for some performance indices. Specifically, we assume constant p for all products and an i.i.d. scaled Poisson random variable D_0 to represent original non-opaque product demands. To obtain the distribution of D_0 , we first start from a Poisson random variable with parameter λ (which we call *base Poisson parameter* of a scaled Poisson random variable) and scale its distribution to make its mean equal to μ . Specifically, denoting $Y_\gamma \sim \text{Pois}(\gamma)$, we can represent D_0 as $D_0 = \frac{\mu}{\lambda} Y_\lambda$. It follows that the fully adjusted demand D_1 under the $(n, 1)$ opaque scheme (i.e., D_p with $p = 1$; a sample mean of n i.i.d. D_0) also follows a scaled Poisson distribution: $D_1 = \frac{\mu}{n\lambda} Y_{n\lambda}$.² For

² Note that D_p with $p \in (0, 1)$ and $n \neq 1$ is not scaled Poisson; it is a complex mixture of two scaled Poisson distributions.

Table 2
Properties of scaled Poisson demands D_0, D_1 , and \bar{D}_1 .

Demand	Base Poisson parameter	Mean	Variance	c_v
D_0	λ	μ	$\sigma_{n,0}^2 = \frac{\mu^2}{\lambda} (=:\sigma^2)$	$\frac{1}{\sqrt{\lambda}}$
D_1	$n\lambda$	μ	$\sigma_{n,1}^2 = \frac{\mu^2}{n\lambda} (= \frac{\sigma^2}{n})$	$\frac{1}{\sqrt{n\lambda}}$
\bar{D}_1	$nm\lambda$	μ	$\frac{\mu^2}{nm\lambda} (= \frac{\sigma^2}{nm})$	$\frac{1}{\sqrt{nm\lambda}}$

use in Proposition 2, we also introduce a sample mean of i.i.d. D_1 over m shelf life periods: $\bar{D}_1 = \frac{\mu}{nm\lambda} Y_{nm\lambda}$. For ease of reference, we summarize the properties of these scaled Poisson demands in Table 2.

3.4. Relative variance $\sigma_{rel}^2(p)$

To find $\sigma_{n,p}^2$, it is convenient to use a new indicator, *relative variance* $\sigma_{rel}^2(p)$. It represents the proximity of $\sigma_{n,p}^2$ to the sample mean variance; see Definition 1. Notice that the minimization of $\sigma_{rel}^2(p)$ is essentially equivalent to minimizing $\sigma_{n,p}^2(p)$. From now on, we will consider $\sigma_{rel}^2(p)$.

Definition 1. For all $n \geq 2$ and $p \in [0, 1]$, relative variance under the (n, p) opaque scheme is defined as

$$\sigma_{rel}^2(p) := \frac{\sigma_{n,p}^2 - \sigma_{n,1}^2}{\sigma_{n,0}^2 - \sigma_{n,1}^2} = \frac{\sigma_{n,p}^2 - \frac{\sigma^2}{n}}{\sigma^2 - \frac{\sigma^2}{n}}. \tag{2}$$

Note that $\sigma_{rel}^2(p)$ takes $\sigma_{rel}^2 = \sigma^2$ when $p = 0$ and takes $\sigma_{rel}^2 = \sigma^2/n$ when $p = 1$ (see Lemma 1). By definition, we observe $\sigma_{rel}^2(p) \in [0, 1]$, $\sigma_{rel}^2(0) = 1$, and $\sigma_{rel}^2(1) = 0$. If we know $\sigma_{rel}^2(p)$ (which we simply denote as σ_{rel}^2 unless there is confusion), we can derive $\sigma_{n,p}^2$ as

$$\sigma_{n,p}^2 = \frac{1 + (n-1)\sigma_{rel}^2}{n} \sigma^2 = \sigma_{n,1}^2 + (n-1)\sigma_{rel}^2 \sigma_{n,1}^2. \tag{3}$$

The new indicator σ_{rel}^2 is introduced primarily for mathematical convenience; however, σ_{rel}^2 has a relationship with a popular indicator, an average correlation coefficient ρ_p of adjusted demands. We observe that a reduction of $\sigma_{rel}^2 (= \sigma_{rel}^2(p))$ makes all adjusted demands closer to their sample mean, and therefore, increases an average correlation coefficient ρ_p of adjusted demands. For example, it is expected that $\sigma_{rel}^2(p) = 1$ and $\rho_p = 0$ at $p = 0$, and $\sigma_{rel}^2(p) = 0$ and $\rho_p = 1$ at $p = 1$. The following lemma tells us the relationship between $\sigma_{rel}^2(p)$ and ρ_p for any $p \in [0, 1]$.

Lemma 2. *Consider the (n, p) opaque scheme, where $n \geq 2$. Relative variance and correlation coefficient satisfy the following relationships.*

$$\sigma_{rel}^2(p) = \frac{1 - \rho_p}{1 + (n-1)\rho_p},$$

or equivalently,

$$\rho_p = \frac{1 - \sigma_{rel}^2(p)}{1 + (n-1)\sigma_{rel}^2(p)}. \tag{4}$$

Remark 1. Lemma 2 indicates that $\sigma_{rel}^2(p)$ and ρ_p satisfy the involution property: The inverse operation and the forward operation are the same.

3.5. Exact and approximate representations of $\sigma_{rel}^2(p)$: $n = 2$ case

We now consider the $n = 2$ case and find the exact analytical expression for σ_{rel}^2 . First, we note the following proposition.

Proposition 1. Consider the (n, p) opaque scheme with $n = 2$. Define a random variable $T = X_p^1 - X_p^2 - X_p^0$ and its variance $\sigma_T = \text{var}(T)$. Relative variance of adjusted demand is exactly represented as a function of T :

$$\sigma_{rel}^2 = \frac{2\mathbb{E}[T^2|T > 0]Pr(T > 0)}{\sigma_T^2}. \quad (5)$$

Eq. (5) in Proposition 1 is exact, but is not convenient when we evaluate σ_{rel}^2 . We show an approximation in Corollary 2.

Corollary 2. Let D_0 follow a scaled Poisson distribution with $c_v = 1/\sqrt{\lambda}$. Denote $\alpha = p\sqrt{2\lambda} = \frac{p}{c_v}\sqrt{2}$. Using the standard normal CDF $\Phi(\cdot)$ and PDF $\phi(\cdot)$, we obtain the following approximation.

$$\sigma_{rel}^2 \approx 2(1 + \alpha^2)\Phi(-\alpha) - 2\alpha\phi(\alpha). \quad (6)$$

Eq. (6) shows that relative variance σ_{rel}^2 is a function of α ($\propto p/c_v$); thus, we know that adjusted variance $\sigma_{n,p}^2$ does not change if both p and c_v proportionally change. This property provides a useful tip for practitioners when implementing our opaque scheme. Specifically, if we observe a change of c_v in product demands, we should change p proportionally in order to maintain the same level of $\sigma_{n,p}^2$ (see Fig. 1c and the discussion therein). Note that Eq. (6) is obtained for $n = 2$; however, it can be used to obtain an approximate value of σ_{rel}^2 for any n since σ_{rel}^2 is not sensitive to n according to our numerical experiment (see Fig. 2b and the discussion therein).

4. Perishable inventory system

In this section, we analyze the effect of our opaque scheme on the total cost of a single product lost-sales perishable inventory system [28]. As in the previous sections, original demands of all non-opaque products are represented by an i.i.d. scaled Poisson random variable D_0 with mean μ ($= \mathbb{E}[D]$) and variance σ^2 ($= \mu^2/\lambda$). We assume all orders are placed at the start of each period to make the inventory level equal to the base-stock level q units; all ordered items arrive instantaneously and are new; and inventory is depleted following a FIFO issuance rule. If any orders are not fulfilled due to a lack of inventory, they are lost and incur a per-unit shortage cost r (sales price minus purchase cost). If any products in inventory are not sold within m shelf life periods, they are discarded and incur a per-unit wastage cost θ (purchase cost plus recycling/landfill/compost cost minus salvage value of an outdated product). At the end of business hours, a seller implements an (n, p) opaque scheme with BPD (i.e., fulfilling opaque orders using non-opaque products with least orders).

Let S and W be the number of shortages (lost sales) and wastages per period per product, respectively. We define the total cost as $C = rS + \theta W$. We consider that the total holding cost is fixed with negligible per-product holding cost, and omit it from the total cost. Our goal is to minimize the expected total cost $\mathbb{E}[C]$ utilizing the (n, p) opaque scheme with BPD.

Denote CDF and PMF of a discrete random variable Y as $F_Y(\cdot)$ and $P_Y(\cdot)$, respectively. Table 2 summarizes the definitions of three scaled Poisson random demands we use: D_0 , D_1 , and \bar{D}_1 . Let $s = \lfloor \frac{n\lambda q}{\mu} \rfloor$ and $Y_\gamma \sim \text{Pois}(\gamma)$. We obtain the following proposition.

Proposition 2. Consider implementing the $(n, 1)$ opaque scheme for perishable products with m -period shelf life. The expected per-period per-product shortages, wastages, and total cost satisfy that, for any base-stock level $q \geq 0$,

$$\mathbb{E}[S] = \mathbb{E}[D_1 - q]^+ = (\mu - q)(1 - F_{Y_{n\lambda}}(s)) + \mu P_{Y_{n\lambda}}(s), \quad (7)$$

$$m\mathbb{E}\left[\frac{q}{m} - \bar{D}_1\right]^+ \geq \mathbb{E}[W] \geq \mathbb{E}\left[\frac{q}{m} - \bar{D}_1\right]^+ = \left(\frac{q}{m} - \mu\right) F_{Y_{nm\lambda}}(s) + \mu P_{Y_{nm\lambda}}(s). \quad (8)$$

Thus, denoting $\bar{C}_{LB} = r\mathbb{E}[D_1 - q]^+ + \theta\mathbb{E}\left[\frac{q}{m} - \bar{D}_1\right]^+$, $\bar{C}_{LB} \leq \mathbb{E}[C] \leq m\bar{C}_{LB}$ holds.

According to Proposition 2, $\mathbb{E}[C] \approx 0$ holds at a wider range of q as $n\lambda$ increases since $F_{Y_{n\lambda}}(s)$ and $F_{Y_{nm\lambda}}(s)$ approach a step function of q at $q = \mu$ and $q = m\mu$, respectively, and both $P_{Y_{n\lambda}}(s)$ and $P_{Y_{nm\lambda}}(s)$ shrink (note: we observe this property experimentally in Figs. 5 and 6). At the limit of $n\lambda \rightarrow \infty$, we observe $D_1, \bar{D}_1 \rightarrow \mu$ and $\mathbb{E}[C] \rightarrow 0$ for any $q \in [\mu, m\mu]$. This insensitivity property was first pointed out by [29]; the property is important for practitioners who prefer their inventory systems to be robust.

Proposition 2 can be used to find the condition to achieve $\mathbb{E}[C] \approx 0$. For this purpose, we introduce threshold parameters n_{th} and σ_{th}^2 as Definition 2. These parameters provide useful information for practitioners trying to achieve $\mathbb{E}[C] \leq m\delta$ (where δ is sufficiently small; see Table 3 and the discussion therein).

Definition 2. For sufficiently small δ , the threshold variance σ_{th}^2 is defined as

$$\sigma_{th}^2 = \max_{n \geq 2} \{\sigma_{n,1}^2 \mid \bar{C}_{LB} \leq \delta\}. \quad (9)$$

Also define the threshold number of non-opaque products as $n_{th} = n$ s.t. $\sigma_{n,1}^2 = \sigma_{th}^2$.

5. Numerical experiments

5.1. Simulation settings

In this section, we numerically examine the effectiveness of our (n, p) opaque scheme. We split the experiments into two parts. In the first part (Sections 5.2 and 5.3), we obtain a series of simulated adjusted demands D_p under the opaque scheme, examine them, and evaluate the impact of the opaque scheme on the variance of adjusted demands $\sigma_{n,p}^2$ (or σ_{rel}^2); in the second part (Sections 5.4 to 5.6), we use the simulated D_p to run a Monte Carlo simulation and evaluate the impact of the opaque scheme on the expected per-period total cost $\mathbb{E}[C]$.

We control variables as follows: the number of non-opaque products $n \in [1, 12]$, proportion $p \in [0, 1]$, base Poisson parameter $\lambda \in \{4, 6, 8, 10, 12, 14\}$ which correspond to coefficient of variation $c_v \in \{0.50, 0.41, 0.35, 0.32, 0.29, 0.27\}$, and average demand $\mu = 10$. We first generate 10,000 (sample values of) original demand D_0 for each one of the n products. We then generate 10,000 sets of intermediate demands X_p^0 and $X_p^i, i \in \{1, \dots, n\}$, from which we obtain 10,000 adjusted demand D_p following BPD. These numerically-obtained D_p are used for a 10,000-period Monte Carlo simulation for a perishable inventory model under a base-stock policy with a base-stock level $q \in [0, 50]$. We allow demands and orders for inventory replenishment to take non-integer values in our experiments. In this numerical study, we only consider shortage and wastage costs when evaluating the expected total cost $\mathbb{E}[C]$, omitting a holding cost from consideration (see the discussion in Section 6.6).

5.2. Impact of c_v of non-opaque products on variance $\sigma_{n,p}^2$

Fig. 1 examines the impact of c_v of the original demand D_0 on $\sigma_{n,p}^2$ when there are 2 non-opaque products ($n = 2$). We observe in Fig. 1a the convergence of adjusted demand variance to the sample mean variance for various c_v values; however, the comparison is not easy due to different variance of original demands with various c_v values. Thus we plot relative variance σ_{rel}^2 in Fig. 1b. Our simulation result indicates that the speed of convergence is strongly affected by c_v of D_0 ; specifically, as c_v gets larger, σ_{rel}^2 converges to 0 more slowly. This property is expected since we need a higher opaque proportion p to suppress the demand variability of D_0 when D_0 is highly variable.

We further investigate this property in Fig. 1c. Here we plot σ_{rel}^2 as a function of p/c_v ; the simulation results for various different parameters all align to the same curve: Eq. (6). This property is useful since Eq. (6) (or Fig. 1c) determines σ_{rel}^2 for any c_v . For example, Fig. 1c indicates $\sigma_{rel}^2 = 0.2$ for $p/c_v = 0.6$. Thus, if $c_v = 0.5$ ($\lambda = 4$), then

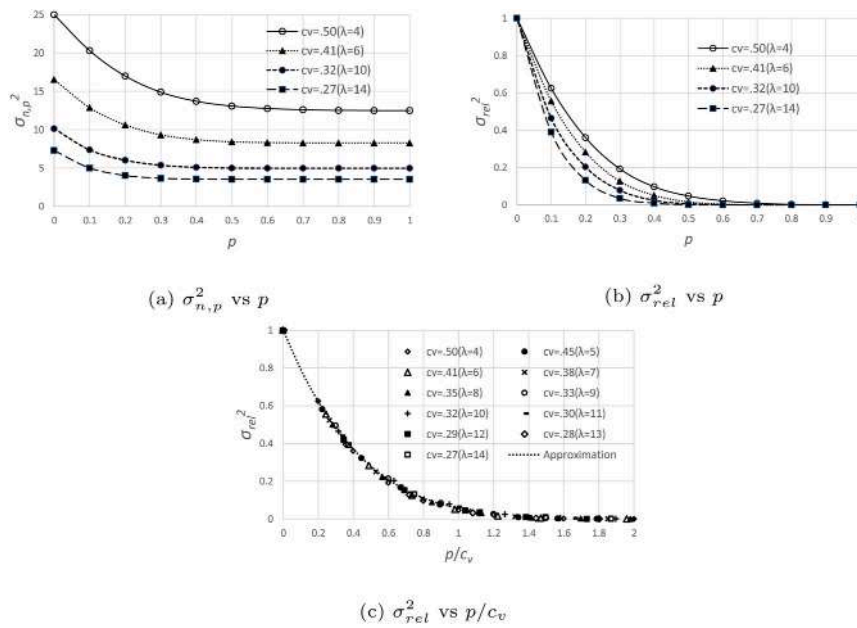


Fig. 1. The effect of c_v (or λ) on $\sigma_{n,p}^2$ and σ_{rel}^2 for $n = 2$.

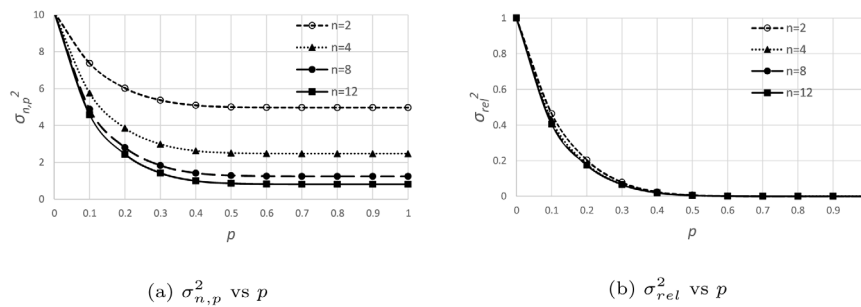


Fig. 2. The effect of n on $\sigma_{n,p}^2$ and σ_{rel}^2 for $c_v = 0.32$ ($\lambda = 10$).

$p = 0.6 \cdot 0.5 = 0.3$ is required to achieve $\sigma_{rel}^2 = 0.2$ (or equivalently, $\sigma_{n,p}^2 = 12.5 + 0.2 \cdot 12.5 = 15$ following Eq. (3) and $\sigma_{n,1}^2 = \sigma^2/n = 12.5$), which is consistent with the result in Fig. 1b. We discuss σ_{rel}^2 only for the $n = 2$ case, but the following result implies that σ_{rel}^2 is insensitive to n .

5.3. Impact of the number of non-opaque products n on variance $\sigma_{n,p}^2$

Fig. 2a examines the impact of the number of non-opaque products $n (\geq 2)$ on $\sigma_{n,p}^2$. To examine the speed of convergence ($\sigma_{n,p}^2 \rightarrow \sigma_{n,1}^2 = \sigma^2/n$ as $p \rightarrow 1$), we fix $c_v = 0.32$ ($\lambda = 10$) and plot σ_{rel}^2 for each n in Fig. 2b. Fig. 2b shows an approximate insensitivity property: σ_{rel}^2 is almost independent from n . Thus, for any n or c_v , we can derive an approximate value of $\sigma_{n,p}^2$ using Eqs. (3) and (6). For example, Fig. 1c indicates that $\sigma_{rel}^2 = 0.2$ at $p/c_v = 0.6$, thus, for $n = 4$ and $c_v = 0.32$ ($\lambda = 10$), we obtain $\sigma_{n,p}^2 = 2.5 + 0.2 \cdot 7.5 = 4$ at $p = 0.6 \cdot 0.32 = 0.2$; this result is consistent with Fig. 2a.

5.4. Impact of the (n, p) opaque scheme on the expected total cost $\mathbb{E}[C]$

In the previous section, we observed that $\sigma_{n,p}^2$ of adjusted demand D_p can be arbitrarily reduced by controlling (n, p) under the opaque scheme. We now examine how D_p and its variance $\sigma_{n,p}^2$ under the (n, p) opaque scheme affect the expected per-period total cost $\mathbb{E}[C] = r\mathbb{E}[S] + \theta\mathbb{E}[W]$. We also evaluate the condition to make $\mathbb{E}[C] \approx 0$.

Fig. 3 shows the impact of p on $\mathbb{E}[S]$ and $\mathbb{E}[W]$ while fixing $c_v = 0.32$ ($\lambda = 10$) and $n = 12$; $\mathbb{E}[S]$ and $\mathbb{E}[W]$ are then combined to obtain $\mathbb{E}[C]$ for $m = 2$ in Fig. 3e and for $m = 3$ in Fig. 3f. We set $r = \theta = 1$ to draw Figs. 3e and 3f. (Note: only the ratio r/θ is important since $\mathbb{E}[C]$ has an arbitrary unit in our plots.) We observe that $\mathbb{E}[C] \approx 0$ for a wider range of q when the proportion p is higher. Note that $\mathbb{E}[S]$ for $m = 2$ and $m = 3$ (Figs. 3a and 3b, respectively) are identical since shortages do not depend on shelf life m , while $\mathbb{E}[W]$ (Figs. 3c and 3d) show strong dependency on m ; larger m reduces $\mathbb{E}[W]$ given q . The optimal base-stock level, which is determined by the trade-off between shortage and wastage, thus depends on m (as well as cost parameters, which we assume $r = \theta = 1$); in this experiment we observe $q = 15$ for $m = 2$ (Fig. 3e) and $q = 18$ for $m = 3$ (Fig. 3f). Since $\mathbb{E}[W]$ is smaller for larger m , the optimal $\mathbb{E}[C]$ is smaller and the optimal base-stock level that minimizes $\mathbb{E}[C]$ is larger for larger m .

Next, we consider some cases with $r \neq \theta$. We set the ratio between shortage and wastage costs to $r/\theta = 2$ (Figs. 4a and 4b) and $r/\theta = 0.5$ (Figs. 4c and 4d) and observe the plots of $\mathbb{E}[C]$. As expected, when $r/\theta > 1$ ($r/\theta < 1$), shortages (wastages, respectively) play more important role than $r/\theta = 1$ case. We continue to observe the benefit of using the opaque scheme: As p increases, $\mathbb{E}[C]$ goes down. Because the property of $\mathbb{E}[C]$ does not change qualitatively (although values change), we use $r = \theta = 1$ in subsequent numerical experiments.

Following the same procedure as above, we vary n while fixing $c_v = 0.32$ ($\lambda = 10$) and $p = 1$, and obtain $\mathbb{E}[C]$ for $m = 2$ in Fig. 5a and for $m = 3$ in Fig. 5b. We observe similarities between Figs. 3e and

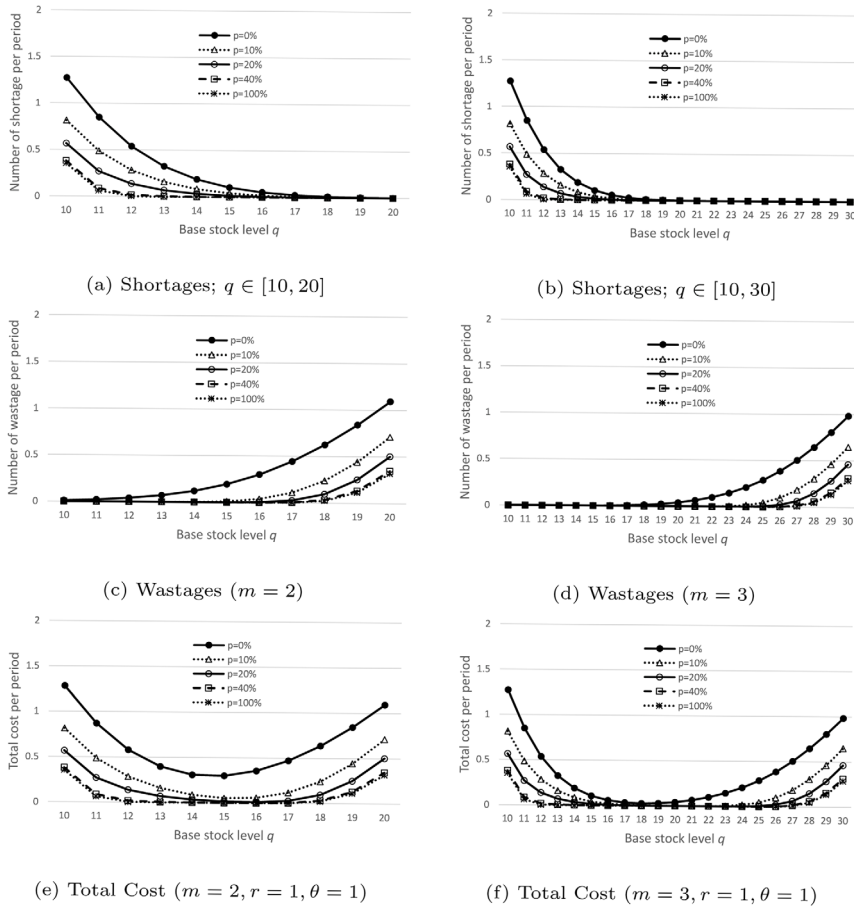


Fig. 3. The effect of p on shortages, wastages, and total cost for $c_v = 0.32$ ($\lambda = 10$), $n = 12$. Notes: Plots in (a) and (b) are equivalent except for the range; we use $[10, 20]$ for $m = 2$ while we use $[10, 30]$ for $m = 3$. Both $p = 0\%$ lines in (e) and (f) correspond to the baseline case with no opaque scheme.

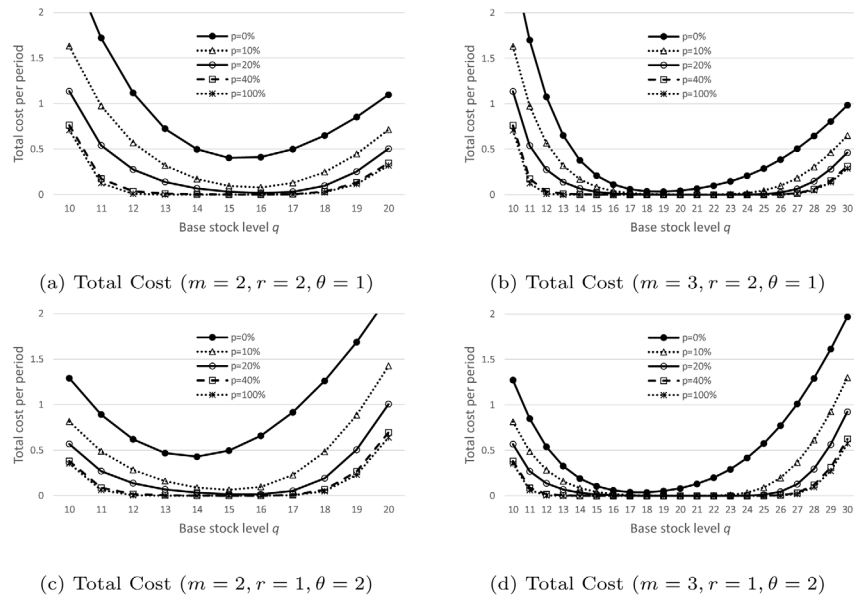


Fig. 4. The effect of r/θ on total cost for $c_v = 0.32$ ($\lambda = 10$), $n = 12$.

5a and between Figs. 3f and 5b. For example, $n = 12, p = 10\%$ lines in Figs. 3e and 3f closely resemble $n = 2, p = 100\%$ lines in Figs. 5a and 5b, respectively; $n = 12, p = 20\%$ and $n = 4, p = 100\%$ lines also look very similar to each other. These similarities are due to the proximity of variances: $\sigma_{12,0.1}^2 \approx \sigma_{2,1}^2$ ($= \sigma^2/2$) and $\sigma_{12,0.2}^2 \approx \sigma_{4,1}^2$ ($= \sigma^2/4$)

hold (see Fig. 2a). This observation implies that the value of $\sigma_{n,p}^2$ is a major determinant factor for $\mathbb{E}[C]$ under the opaque scheme. (Note: we observe this property more explicitly in Fig. 7.)

We observe a diminishing return property for the (n, p) opaque scheme in Figs. 3 and 5: the marginal impact on $\mathbb{E}[C]$ is larger when p

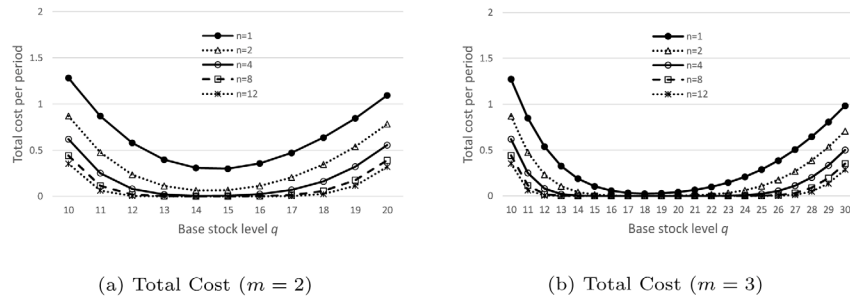


Fig. 5. The effect of n on total cost for $c_p = 0.32$ ($\lambda = 10$), $p = 1$. Notes: This figure and all other figures below assume $r = \theta = 1$. Both $n = 1$ lines in (a) and (b) correspond to the baseline case with no opaque scheme.

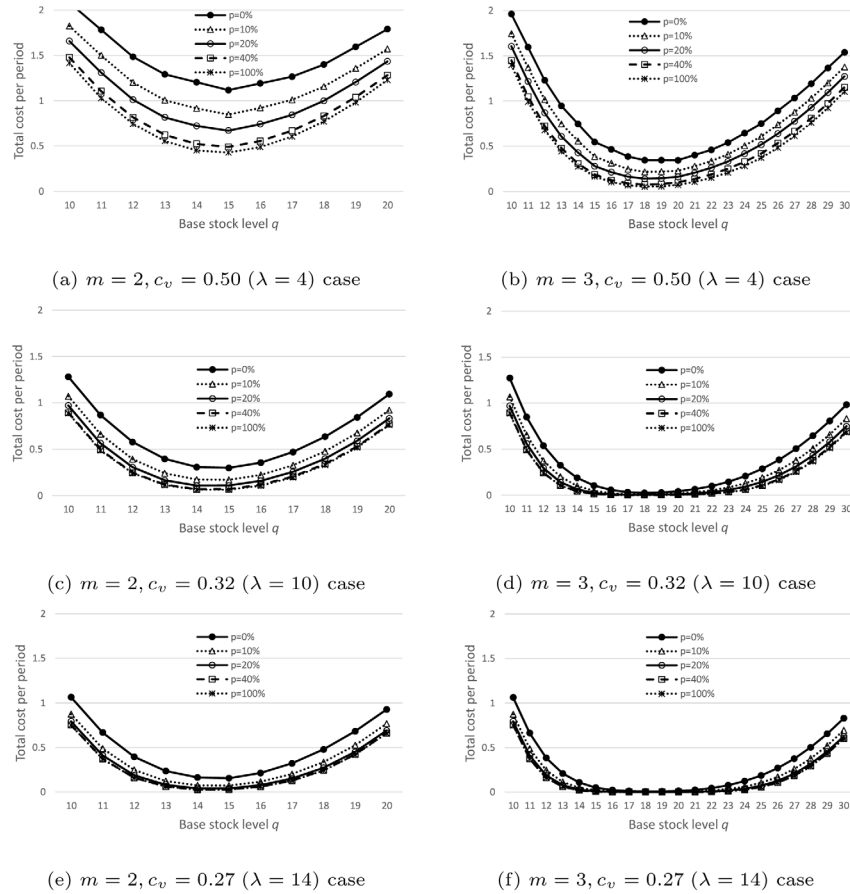


Fig. 6. Effect of c_v on $\mathbb{E}[C]$ for $n = 2$.

or n are smaller. For example, a change from $p = 0\%$ to 10% in Fig. 3e and a change from $n = 1$ to 2 in Fig. 5a reduce $\mathbb{E}[C]$ most (compared to other changes such as a change from $p = 10\%$ to 20%). This is also attributed to the dependence of $\mathbb{E}[C]$ on $\sigma_{n,p}^2$: We observe a larger marginal reduction of $\sigma_{n,p}^2$ when p or n are smaller according to Fig. 2a.

5.5. Impact of c_v of non-opaque products on the expected total cost $\mathbb{E}[C]$

We next examine the impact of variability (c_v) of original demand D_0 on $\mathbb{E}[C]$. In Fig. 6, we use $c_v \in \{0.50, 0.32, 0.27\}$ for $n = 2$ and compare their results. First, we observe that $\mathbb{E}[C]$ is closer to zero for smaller c_v given q ; this observation is attributed to the smaller $\sigma_{n,p}^2$ for smaller c_v (see also Fig. 1a). Second, $\mathbb{E}[C]$ converges faster to its minimum for smaller c_v as $p \rightarrow 1$; this observation is attributed to the increased speed of convergence of $\sigma_{n,p}^2$ to its minimum $\sigma_{n,1}^2$ for smaller c_v (see also Fig. 1b). These results imply that the (n, p) opaque scheme impacts $\mathbb{E}[C]$ mainly through the change in the variance $\sigma_{n,p}^2$ of D_p .

5.6. Impact of variance $\sigma_{n,p}^2$ on the expected total cost $\mathbb{E}[C]$

So far, our experiments indicate a strong dependence of $\mathbb{E}[C]$ on $\sigma_{n,p}^2$. To observe the relationship between $\sigma_{n,p}^2$ and $\mathbb{E}[C]$ more explicitly, we plot them in Fig. 7. In this figure, we fix q around its optimal base-stock level and vary $n \in \{2, 4, 8, 12\}$ and $p \in \{0, 0.1, 0.2, \dots, 1\}$ for either $m = 2$ or $m = 3$ cases. Fig. 7 shows that $\mathbb{E}[C]$ is approximately linearly dependent on $\sigma_{n,p}^2$ irrespective of (n, p) except when $\mathbb{E}[C] \approx 0$. This result indicates that the reduction of $\mathbb{E}[C]$ is primarily driven by the reduction of $\sigma_{n,p}^2$ until $\mathbb{E}[C]$ becomes close to zero; once we achieve $\mathbb{E}[C] \approx 0$, the further reduction of $\sigma_{n,p}^2$ does not impact $\mathbb{E}[C]$. The threshold values of $\sigma_{n,p}^2$ that achieve $\mathbb{E}[C] \approx 0$ would provide a useful tip for practitioners: it is not worthwhile to reduce $\sigma_{n,p}^2$ beyond the threshold since the maximum cost saving is already achieved. However, the exact threshold is hard to obtain since it depends on both n and p ;

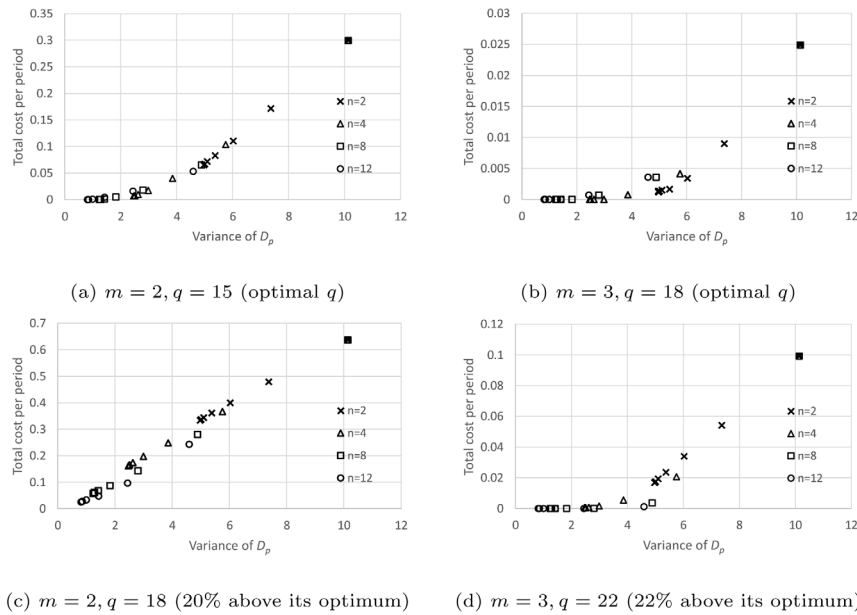


Fig. 7. Effect of $\sigma_{n,p}^2$ on $\mathbb{E}[C]$ for $c_v = 0.32$ ($\lambda = 10$). Notes: Eleven plots corresponding to $p = 0\%$ to 100% for each n are shown in each figure. Solid square dots on the top right corner of all figures correspond to the $p = 0\%$ case for all n (baseline case with no opaque scheme).

Table 3
Comparison between $\mathbb{E}[C]$ and \overline{C}_{LB} .

m	q	n	1	2	4	8	12
		$\sigma_{n,1}^2 = 10/n$	10	5	2.5	1.25	0.83
2	15	$\mathbb{E}[C]$	0.2993	0.0673	0.0067	0.0002	0.0000
		\overline{C}_{LB}	0.2287	0.0465	<u>0.0042</u>	0.0001	0.0000
		$2\overline{C}_{LB}$	0.4574	0.0930	0.0084	0.0002	0.0000
	18	$\mathbb{E}[C]$	0.6365	0.3455	0.1610	0.0577	0.0249
		\overline{C}_{LB}	0.4759	0.2434	0.1080	0.0363	0.0156
		$2\overline{C}_{LB}$	0.9519	0.4868	0.2161	0.0726	0.0311
3	18	$\mathbb{E}[C]$	0.0249	0.0006	0.0000	0.0000	0.0000
		\overline{C}_{LB}	0.0183	<u>0.0005</u>	0.0000	0.0000	0.0000
		$3\overline{C}_{LB}$	0.0549	0.0016	0.0000	0.0000	0.0000
	22	$\mathbb{E}[C]$	0.0993	0.0166	0.0006	0.0000	0.0000
		\overline{C}_{LB}	0.0469	<u>0.0065</u>	0.0003	0.0000	0.0000
		$3\overline{C}_{LB}$	0.1407	0.0194	0.0008	0.0000	0.0000

Notes: $c_v = 0.32$ ($\lambda = 10$), $p = 1$. $q = 15$ is optimal when $m = 2$. $q = 18$ is optimal when $m = 3$. The underlined values specify that \overline{C}_{LB} is the largest value that goes below $\delta = 0.01$.

alternatively, we can approximate the exact threshold by σ_{th}^2 , which is determined by Eq. (9) in Definition 2.

Table 3 shows the total cost $\mathbb{E}[C]$ and its theoretical lower and upper bounds \overline{C}_{LB} and $m\overline{C}_{LB}$, respectively, as implied by Proposition 2. We evaluate σ_{th}^2 using sufficiently small δ ($\delta = 0.01$). By observing the values of \overline{C}_{LB} in Table 3, it is straightforward to identify the smallest n and the largest $\sigma_{n,1}^2$ that satisfy $\overline{C}_{LB} \leq \delta$ (and thus $\mathbb{E}[C] \leq m\overline{C}_{LB} \leq m\delta$): We obtain $\sigma_{th}^2 = 2.5$ at $q = 15$ and $\sigma_{th}^2 < 0.83$ at $q = 18$ for $m = 2$; and $\sigma_{th}^2 = 5$ at both $q = 18$ and 22 for $m = 3$. These rough estimates of σ_{th}^2 are consistent with the thresholds observed in Fig. 7.

6. Discussion and managerial insights

In this section, we discuss various issues and managerial insights of our opaque scheme in detail. For readers' convenience, we list key questions one by one below.

6.1. Input parameters used in this study

Our model has several input parameters. An average product demand is set to $\mu = 10$ and per-period shortage and wastage costs are set to $s, \theta \in \{1, 2\}$; these parameters have arbitrary units and, thus, are applicable to various cases (e.g., lb, kg, \$, €). The ratio s/θ affects the shape of the cost curve (see Fig. 4); however, the fundamental properties we obtain do not change as discussed in Section 5.4. Two important input parameters are a product coefficient of variation and product shelf life periods; we set them as $c_v \in [0.27, 0.5]$ and $m \in \{2, 3\}$ to reflect the commonly observed values (for example, c_v ranges from 0.1 to 0.5 in [30], and shelf lives of baked goods, cut fruits/vegetables, and prepared foods sold at retail stores are around two to three days). If $c_v > 0.5$, the effectiveness of the opaque scheme is even greater than the results we obtain in this study. In contrast, if c_v is very small (e.g. $c_v \lesssim 0.1$), there is almost no benefit for implementing an opaque scheme because of very small demand variability. Likewise, if $m > 3$, products can be treated almost the same as non-perishables since the amount of wastage is very small; our opaque scheme does not provide any advantage for such cases.

6.2. Benefit of our opaque scheme

Our opaque scheme aims to provide a new method to help retailers better manage their inventory systems when suffering from large shortages and wastages due to high variability of product demand. Our opaque scheme using BPD is effective for perishables: It reduces the variability of product demands, reduces shortages and wastages of perishables, and achieves a lower total cost (a higher cost saving). We provide rules of thumb to determine necessary parameters for the implementation of our opaque scheme. Customers also benefit from our opaque scheme. An opaque item for customers is simply an added option; if they do not want to purchase an opaque product, they do not need to. However, if they do purchase an opaque product, they may benefit from a discount or reward such as free/reduced shipping fee. Finally, and most importantly, the society would benefit from the opaque scheme for a potentially large aggregate reduction of food wastages. The opaque scheme, if implemented by many retail stores, could help the society to contribute to the Sustainable Development Goals (SDG) Target 12.3: "By 2030, halve per capita global food waste at the retail and consumer levels and reduce food losses along production and supply chains, including post-harvest losses" [31].

6.3. Recommendations for managers

When product demands are highly variable, managers could first check the threshold number of non-opaque products (Definition 2) needed to achieve the maximum total cost saving. Assuming that the number of non-opaque products available for them is less than this threshold, retail store managers may next refer to the approximate relative variance (Corollary 2) to obtain the approximate p necessary to achieve most of the benefit provided by the opaque scheme. For example, we show in Fig. 1c that 80% of the benefit of the opaque scheme is achieved by setting $p = 0.6c_v$, or 90% of the benefit of opaque scheme is achieved by setting $p = 0.8c_v$, where the benefit is defined as the reduction of variance, leading to the expected total cost savings. Since c_v is often much less than 1 (see, e.g., [30]), a smaller p is sufficient to capture most of the benefit of the opaque scheme.

6.4. Ease of implementation in practice

The key factor that needs to be warranted is the anonymity of an opaque product: its specification must be opaque. Thus, a traditional in-person sales (where customers can easily see what is sold as an opaque product) is less suitable for an opaque selling scheme (if not impossible). This situation has drastically changed due to the rapid rise of mobile apps (E-commerce via internet enabled phones). Through mobile apps, customers make orders and complete payments from within the store or outside of the store; after orders are made via mobile apps, customers pick up ordered items (including opaque products) from staff without going through a cash register. This scheme is gaining popularity since it saves time for both customers and staff members. An opaque sales can be implemented with almost no extra cost if stores already adopt mobile apps.

6.5. Obstacles for implementation

Although our BPD opaque scheme is simple and easy to implement in most cases, there are some cases that make the implementation of BPD challenging. First, we require some portion of customers to be indifferent among non-opaque products. If all customers have a strong preference to purchase a specific (non-opaque) product or if only a few switch from non-opaque to opaque products when the discount for an opaque product is huge, opaque schemes do not work well. The second, possibly more serious, obstacle for the opaque scheme is the positive correlation among all non-opaque product demands. If strong positive correlations exist among non-opaque product demands, an opaque demand (pooled demand) cannot effectively reduce the variability of non-opaque product demands. This same issue is called systematic risk (or market risk) in the insurance and finance fields; a risk pooling scheme does not work in this situation.

6.6. Limitations of our opaque scheme

The current study has many simplifying assumptions: scaled Poisson and i.i.d. non-opaque product demands, a constant probability to shift from each non-opaque product to an opaque product, and a constant total customer demand before/after the implementation of the opaque scheme. The inventory model we consider is also restricted, as we only consider the zero lead time case. We also assume that a variable (per-unit) holding cost is negligible compared to the fixed holding cost and shortage/wastage costs; this assumption is reasonable when a facility cost (including utility costs such as electricity and heating costs) takes a large portion of the total holding cost. Finally, we simplify the analysis in this paper by limiting our discussion only on BPD; however, in practice, a hybrid of BPD and BPI may be a better alternative. In fact, many extensions to our model are necessary to further investigate the effectiveness of opaque schemes. We will address these extensions in the future study.

7. Conclusions

In this paper we studied the effectiveness of an opaque scheme in perishable inventory systems. We showed that our (n, p) opaque scheme based on the balancing policy on demand (BPD) can effectively reduce the variability of orders for perishable products, leading to smaller shortages and wastages in perishable inventory systems. Our study has derived various analytical formulas, which could help practitioners implement our opaque scheme. One of the most important results in this study is Eq. (6) in Corollary 2, which revealed the relationship between the opaque demand proportion p and the coefficient of variation of non-opaque product demands c_v ; this relationship indicates that the ratio p/c_v plays a critical role in determining the reduction of the variability as well as the total cost (shortage and wastage costs). We also provided a convenient indicator, the threshold variance, that achieves the maximum total cost savings from the opaque scheme; this information helps practitioners to decide whether they want to implement our opaque scheme, and if so, which combination of parameters should be used.

We observed that a growing number of retail stores have started to implement various opaque schemes as a tool (on their mobile apps) to reduce food wastage and to make more profit. This study has demonstrated and confirmed that an opaque scheme is effective for perishable inventory systems when the variability of product demands is high. However, it requires further investigation to accommodate real-world complexities. With this study of an opaque scheme for perishable inventory systems, we hope to contribute to the reduction of food wastage, one of the most important sustainable development goals, as well as to encourage further investigations of this promising idea.

CRedit authorship contribution statement

Katsunobu Sasanuma: Conceptualization, Formulation, Data curation, Formal analysis, Writing – original draft. **Akira Hibiki:** Conceptualization, Formulation, Writing – review & editing. **Thomas Sexton:** Conceptualization, Formulation, Writing – review & editing.

Declaration of competing interest

One or more of the authors of this paper have disclosed potential or pertinent conflicts of interest, which may include receipt of payment, either direct or indirect, institutional support, or association with an entity in the biomedical field which may be perceived to have potential conflict of interest with this work. For full disclosure statements refer to <https://doi.org/10.1016/j.orp.2021.100220>. Katsunobu Sasanuma reports financial support was provided by Tohoku University. Katsunobu Sasanuma reports a relationship with Tohoku University that includes: funding grants and travel reimbursement.

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Table A.1
Summary of notations.

Notation	Description
n	Number of non-opaque products
i	Index for non-opaque product $i \in [1, n]$
p_i	Probability to switch from non-opaque product i to opaque product
p	p_i when all p_i are identical
\mathbf{p}	Probability vector representing all p_i
D_0^i	Original demand for non-opaque product i per period
D_0	D_0^i when all D_0^i are i.i.d.
λ	Base Poisson rate for D_0
μ^i	Average original demand for non-opaque product i (expected value of D_0^i)
μ	μ^i when all μ^i are identical
X_p^i	Intermediate demand for non-opaque product i per period
X_p^0	Intermediate demand for opaque product per period
D_p^i	Adjusted demand for non-opaque product i per period
D_p	D_p^i when all D_p^i are i.i.d.
σ^2	Average variance of original demands $D_0^i, \forall i$
$\sigma_{n,p}^2$	Average variance of adjusted demands $D_p^i, \forall i$
$\sigma_{rel}^2(p)$	Relative variance
c_v	Coefficient of variation for non-opaque product
m	Product shelf life periods
r	Shortage cost per item (adjusted by the item cost)
θ	Wastage cost per item (adjusted by the item cost)
S	Number of shortages per period
W	Number of wastages per period
C	Total system cost per period
$F_Y(\cdot)$	Cumulative distribution function (CDF)
$P_Y(\cdot)$	Probability mass function (PMF)

Table A.2
Summary of units of measurement.

Parameter	Unit of measurement
$\mu^i, \mu, D_0^i, D_0, X_p^i, X_p^0, D_p^i, D_p, S, W$	Amount (e.g., mass, volume, or number of items) per period
r, θ	Cost (e.g., US\$, €) per item
C	Cost (e.g., US\$, €) per period

Appendix A. Notation

Tables A.1 and A.2 show the notations and units of measurement, respectively.

Appendix B. Proofs

Proof of Lemma 1. The first inequality, $\sigma^2 \geq \sigma_{n,p}^2$, holds as the opaque scheme is only used to reduce $\sigma_{n,p}^2$; the equality holds when $p = 0$. The second inequality, $\sigma_{n,p}^2 \geq \frac{\sigma^2}{n}$, holds since

$$\begin{aligned} \sigma_{n,p}^2 &= \frac{1}{n} \sum_i \text{var}(D_p^i) = \frac{1}{n} \sum_i \mathbb{E}[(D_p^i - \mu^i)^2] = \mathbb{E} \left[\frac{1}{n} \sum_i (D_p^i - \mu^i)^2 \right] \\ &= \mathbb{E} \left[\left(\frac{1}{n} \sum_i (D_p^i - \mu^i) \right)^2 + \frac{1}{n^2} \sum_{i < j} \sum_j ((D_p^i - \mu^i) - (D_p^j - \mu^j))^2 \right] \\ &\geq \mathbb{E} \left[\left(\frac{1}{n} \sum_i (D_p^i - \mu^i) \right)^2 \right] = \mathbb{E} \left[\left(\frac{1}{n} \sum_i D_p^i - \frac{1}{n} \sum_i \mu^i \right)^2 \right] \\ &= \mathbb{E} \left[\left(\frac{1}{n} \sum_i D_0^i - \frac{1}{n} \sum_i \mu^i \right)^2 \right] = \text{var} \left(\frac{1}{n} \sum_i D_0^i \right) = \frac{\sigma^2}{n}, \end{aligned}$$

where an equality holds when $D_p^i - \mu_i = D_p^j - \mu_j$ for all i and j , which is realized when $p = 1$. \square

Proof of Lemma 2. We use $\sigma_{n,0}^2$ to represent $\sigma_{n,p}^2$ and $\sigma_{n,1}^2$, from which we obtain $\sigma_{rel}^2(p)$.

$$\begin{aligned} \sigma_{n,0}^2 &= \frac{1}{n} \sum_i \text{var}(D_0^i) = \frac{1}{n} \text{var} \left(\sum_i D_0^i \right) = \frac{1}{n} \text{var} \left(\sum_i D_p^i \right) \\ &= \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n \text{cov}(D_p^i, D_p^j) \\ &= \frac{1}{n} \sum_i \text{var}(D_p^i) + \frac{2}{n} \sum_{1 \leq i < j \leq n} \text{cov}(D_p^i, D_p^j) \\ &= (1 + (n-1)\rho_p) \sigma_{n,p}^2, \\ \sigma_{n,1}^2 &= \frac{1}{n} \sigma_{n,0}^2, \\ \sigma_{rel}^2(p) &= \frac{\sigma_{n,p}^2 - \sigma_{n,1}^2}{\sigma_{n,0}^2 - \sigma_{n,1}^2} = \frac{\frac{\sigma_{n,0}^2}{1+(n-1)\rho_p} - \frac{\sigma_{n,0}^2}{n}}{\sigma_{n,0}^2 - \frac{\sigma_{n,0}^2}{n}} = \frac{1 - \rho_p}{1 + (n-1)\rho_p}. \quad \square \end{aligned}$$

Proof of Proposition 1. The following identity relationship exists when $n = 2$: $X_p^1 + X_p^2 + X_p^0 = D_p^1 + D_p^2 = D_0^1 + D_0^2$, where X_p^1, X_p^2, X_p^0 are independent scaled Poisson random variables, and D_0^1 and D_0^2 are independent scaled Poisson random variables. Thus, we obtain $\sigma_T^2 = \text{var}(D_p^1 + D_p^2) = \text{var}(D_0^1 \pm D_0^2)$. Since $D_p^1 = D_p^2$ when $p = 1$, we have

$$\begin{aligned} \sigma_{n,p}^2 &= \frac{\text{var}(D_p^1) + \text{var}(D_p^2)}{2} = \frac{\text{var}(D_p^1 + D_p^2)}{4} + \frac{\text{var}(D_p^1 - D_p^2)}{4} \\ &= \frac{\sigma_T^2}{4} + \frac{\text{var}(D_p^1 - D_p^2)}{4}, \\ \sigma_{n,0}^2 &= \frac{\sigma_T^2}{2}, \quad \sigma_{n,1}^2 = \frac{\sigma_T^2}{4}, \quad \sigma_{rel}^2 = \frac{\sigma_{n,p}^2 - \sigma_{n,1}^2}{\sigma_{n,0}^2 - \sigma_{n,1}^2} = \frac{\text{var}(D_p^1 - D_p^2)}{\sigma_T^2}. \end{aligned}$$

Note that D_p^1 and D_p^2 are not independent except when $p = 0$. Thus, to find $\text{var}(D_p^1 - D_p^2)$, we separate the entire state space into three disjoint sets:

$$\begin{aligned} A_1 &= \{X_p^1, X_p^2, X_p^0 \mid X_p^1 - X_p^2 > X_p^0\} = \{X_p^1, X_p^2, X_p^0 \mid T > 0\}, \\ A_2 &= \{X_p^1, X_p^2, X_p^0 \mid |X_p^1 - X_p^2| \leq X_p^0\}, \\ A_3 &= \{X_p^1, X_p^2, X_p^0 \mid X_p^1 - X_p^2 < -X_p^0\}. \end{aligned}$$

Here, observe that A_1 and A_3 are identical when X_p^1 and X_p^2 are flipped, and also that $D_p^1 - D_p^2 = T$ given A_1 and $D_p^1 - D_p^2 = 0$ given A_2 . Therefore we have

$$\begin{aligned} \text{var}(D_p^1 - D_p^2 | A_1) &= \text{var}(D_p^1 - D_p^2 | A_3) = \text{var}(T | T > 0), \\ \text{var}(D_p^1 - D_p^2 | A_2) &= 0, \\ \mathbb{E}[D_p^1 - D_p^2 | A_1] &= -\mathbb{E}[D_p^1 - D_p^2 | A_3] = \mathbb{E}[T | T > 0], \quad \mathbb{E}[D_p^1 - D_p^2 | A_2] = 0, \\ \mathbb{E}[D_p^1 - D_p^2] &= 0, \quad \text{Pr}(A_1) = \text{Pr}(A_3) = \text{Pr}(T > 0). \end{aligned}$$

We can now represent $\text{var}(D_p^1 - D_p^2)$ using T as follows:

$$\begin{aligned} \text{var}(D_p^1 - D_p^2) &= \mathbb{E}[\text{var}(D_p^1 - D_p^2 | A)] + \text{var}(\mathbb{E}[D_p^1 - D_p^2 | A]) \\ &= \sum_i \text{var}(D_p^1 - D_p^2 | A_i) \text{Pr}(A_i) + \sum_i (\mathbb{E}[D_p^1 - D_p^2 | A_i])^2 \text{Pr}(A_i) \\ &= 2 [\text{var}(T | T > 0) + (\mathbb{E}[T | T > 0])^2] \text{Pr}(T > 0) \\ &= 2 \mathbb{E}[T^2 | T > 0] \text{Pr}(T > 0), \end{aligned}$$

from which we can derive the representation of σ_{rel}^2 . \square

Proof of Corollary 2. The difference between two independent Poisson random variables (or scaled variables of them) is known to follow a Skellam distribution, which is approximately represented by a normal distribution. Since X_p^1 and $X_p^2 + X_p^0$ are independent scaled Poisson random variables, the difference T approximately follows a normal distribution: $T \propto N(-2p\mu, \frac{2\mu^2}{\lambda})$, whose mean is $\mu_T = -2p\mu$ and standard deviation is $\sigma_T = \mu \sqrt{\frac{2}{\lambda}}$. The z-score at $T = 0$ is

$\alpha = \frac{-\mu_T}{\sigma_T} = p\sqrt{2\lambda} = \frac{p}{c_v}\sqrt{2}$. Let also $Z = 1 - \Phi(\alpha) = \Phi(-\alpha) (\approx Pr(Z > 0))$.

The distribution of T given $T > 0$ is approximated by a truncated normal distribution, whose mean and variance are

$$\mathbb{E}[T|T > 0] \approx \mu_T + \sigma_T \frac{\phi(\alpha)}{Z} = \sigma_T \left(-\alpha + \frac{\phi(\alpha)}{Z} \right),$$

$$var(T|T > 0) \approx \sigma_T^2 \left(1 + \alpha \frac{\phi(\alpha)}{Z} - \left(\frac{\phi(\alpha)}{Z} \right)^2 \right).$$

Using these relationships with Proposition 1, we obtain

$$\sigma_{rel}^2 = \frac{2 [var(T|T > 0) + (\mathbb{E}[T|T > 0])^2] Pr(T > 0)}{\sigma_T^2}$$

$$\approx 2(1 + \alpha^2)\Phi(-\alpha) - 2\alpha\phi(\alpha). \quad \square$$

Proof of Proposition 2. We first prove Eq. (7). By definition, $\mathbb{E}[S] = \mathbb{E}[D_1 - q]^+$. Also note that $D_1 = \frac{\mu}{n\lambda} Y_{n\lambda}$ and $P_{Y_{n\lambda}}(s) = \frac{\gamma^s e^{-\gamma}}{s!}$. It is straightforward to obtain

$$\mathbb{E}[S] = \mathbb{E}[D_1 - q]^+ = \mathbb{E}\left[\frac{\mu}{n\lambda} Y_{n\lambda} - q\right]^+ = \frac{\mu}{n\lambda} \mathbb{E}\left[Y_{n\lambda} - \frac{n\lambda q}{\mu}\right]^+$$

$$= \frac{\mu}{n\lambda} \sum_{y=\frac{n\lambda q}{\mu}+1}^{\infty} \left(y - \frac{n\lambda q}{\mu}\right) P_{Y_{n\lambda}}(y)$$

$$= \frac{\mu}{n\lambda} \left[\sum_{y=\frac{n\lambda q}{\mu}+1}^{\infty} y P_{Y_{n\lambda}}(y) - \frac{n\lambda q}{\mu} \sum_{y=\frac{n\lambda q}{\mu}+1}^{\infty} P_{Y_{n\lambda}}(y) \right]$$

$$= \frac{\mu}{n\lambda} \left[n\lambda \sum_{y=\frac{n\lambda q}{\mu}+1}^{\infty} P_{Y_{n\lambda}}(y) - \frac{n\lambda q}{\mu} \sum_{y=\frac{n\lambda q}{\mu}+1}^{\infty} P_{Y_{n\lambda}}(y) \right]$$

$$= \mu \left(1 - F_{Y_{n\lambda}}(s) + P_{Y_{n\lambda}}(s) \right) - q \left(1 - F_{Y_{n\lambda}}(s) \right)$$

$$= (\mu - q) \left(1 - F_{Y_{n\lambda}}(s) \right) + \mu P_{Y_{n\lambda}}(s).$$

Next, we prove Eq. (8). The proof of $\mathbb{E}[W] \geq \mathbb{E}\left[\frac{q}{m} - \bar{D}_1\right]^+$ is presented in [29]; this lower bound is obtained by evaluating the per-period (i.e., $1/m$ of the) expected wastage of the first m periods assuming that all inventory in the beginning of period 1 is fresh. In contrast, the upper bound is the expected wastage of the m th period assuming again that all inventory in the beginning of period 1 is fresh, since the amount of the oldest units is largest in the beginning of the m th period. We omit the remaining part of the derivation of Eq. (8) since it is almost identical to the derivation of Eq. (7) except we use $Y_{nm\lambda}$ instead of $Y_{n\lambda}$. \square

References

[1] Gustavsson J, Cederberg C, Sonesson U, van Otterdijk R, Meybeck A. Global food losses and food waste. Food and Agriculture Organization (FAO) of the United Nations; 2011, Accessed May 25, 2021. URL <http://www.fao.org/3/i2697e/i2697e.pdf>.

[2] Buzby JC, Farah-Wells H, Hyman J. The estimated amount, value, and calories of postharvest food losses at the retail and consumer levels in the United States. In: USDA-ERS economic information bulletin, Vol. 121. 2014, Accessed May 25, 2021. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2501659.

[3] Munesue Y, Masui T, Fushima T. The effects of reducing food losses and food waste on global food insecurity, natural resources, and greenhouse gas emissions. *Environ Econ Policy Stud* 2015;17(1):43–77.

[4] Brodersen R, McDermott C, Elliott D, Moreno L, Mulder C. Oregon wasted food study. Community Environmental Services, Portland State University; 2019.

[5] Sviokla J. Value poaching. *Across Board* 2003;40(2):11–2.

[6] Davis T. Effective supply chain management. *Sloan Manage Rev* 1993;34:35.

[7] Graves SC, Tomlin BT. Process flexibility in supply chains. *Manage Sci* 2003;49(7):907–19.

[8] Simchi-Levi D, Wei Y. Understanding the performance of the long chain and sparse designs in process flexibility. *Oper Res* 2012;60(5):1125–41.

[9] Fay S. Selling an opaque product through an intermediary: The case of disguising one’s product. *J Retail* 2008;84(1):59–75.

[10] Jiang Y. Price discrimination with opaque products. *J Revenue Pricing Manage* 2007;6(2):118–34.

[11] Fay S, Xie J. Probabilistic goods: A creative way of selling products and services. *Mark Sci* 2008;27(4):674–90.

[12] Jerath K, Netessine S, Veeraraghavan SK. Revenue management with strategic customers: Last-minute selling and opaque selling. *Manage Sci* 2010;56(3):430–48.

[13] Anderson CK. Setting prices on priceline. *Interfaces* 2009;39(4):307–15.

[14] Anderson CK, Xie X. A choice-based dynamic programming approach for setting opaque prices. *Prod Oper Manage* 2012;21(3):590–605.

[15] Elmachtoub AN, Hamilton ML. The power of opaque products in pricing. *Manage Sci Artic Adv* 2021;1–17.

[16] Elmachtoub AN, Wei Y, Zhou Y. Retailing with opaque products. 2015, SSRN 2659211. Accessed May 25, 2021. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2659211.

[17] Elmachtoub AN, Yao D, Zhou Y. The value of flexibility from opaque selling. 2019, SSRN 3483872. Accessed May 25, 2021. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3483872.

[18] McGavin EJ, Ward JE, Schwarz LB. Balancing retailer inventories. *Oper Res* 1997;45(6):820–30.

[19] Dominey M, Hill R. Performance of approximations for compound Poisson distributed demand in the newsboy problem. *Int J Prod Econ* 2004;92(2):145–55.

[20] Babai MZ, Jemai Z, Dallery Y. Analysis of order-up-to-level inventory systems with compound Poisson demand. *Eur J Oper Res* 2011;210(3):552–8.

[21] Bohm G, Zech G. Statistics of weighted Poisson events and its applications. *Nucl Instrum Methods Phys Res A* 2014;748:1–6.

[22] Karaesmen IZ, Scheller-Wolf A, Deniz B. Managing perishable and aging inventories: review and future research directions. In: *Planning production and inventories in the extended enterprise*. Springer; 2011, p. 393–436.

[23] Baron O. Managing perishable inventory. In: *Wiley encyclopedia of operations research and management science*. Wiley Online Library; 2011.

[24] Sasanuma K, Delasay M, Pitocco C, Scheller-Wolf A, Sexton T. Marginal analysis of perishable inventory models using the externality principle. 2020, arXiv: 2006.06912. Accessed May 25, 2021. URL <https://arxiv.org/abs/2006.06912>.

[25] Huh WT, Janakiraman G, Muckstadt JA, Rusmevichientong P. Asymptotic optimality of order-up-to policies in lost sales inventory systems. *Manage Sci* 2009;55(3):404–20.

[26] Bijvank M, Huh WT, Janakiraman G, Kang W. Robustness of order-up-to policies in lost-sales inventory systems. *Oper Res* 2014;62(5):1040–7.

[27] Bu J, Gong X, Chao X. Asymptotic optimality of base-stock policies for perishable inventory systems. 2019, Preprint.

[28] Nahmias S, Pierskalla WP. Optimal ordering policies for a product that perishes in two periods subject to stochastic demand. *Nav Res Logist Q* 1973;20(2):207–29.

[29] Chazan D, Gal S. A Markovian model for a perishable product inventory. *Manage Sci* 1977;23(5):512–21.

[30] Nandakumar P, Morton TE. Near myopic heuristics for the fixed-life perishability problem. *Manage Sci* 1993;39(12):1490–8.

[31] Goal 12: Ensure sustainable consumption and production patterns. United Nations Statistics Division (UNSD); 2021, Accessed May 25, 2021. URL <https://unstats.un.org/sdgs/report/2021/goal-12/>.